# Improvement in Doppler Navigator Performance through Spectral Compression

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**A Doppler navigation radar for space applications must be at least 30 db more sensitive than present Doppler sensors but still maintain the same (or better) error tolerances. Yet, solution of the problem (if at all attainable) by conventional approaches is severely hampered by the prohibitive space and power limitations of present spacecraft. This paper will show that proper frequency modulation of the microwave transmitter can code the ground scatterer return in such a way that considerable signal bandwidth reduction (40–50 times) can be effected [referenced to the normal continuous-wave (CW) bandwidth] with an attendant increase in peak spectral power and a reduction of certain critical error sources.**

## Nomenclature

$a_i$ = position of the $i$th scatterer at time $t = 0$
$C$ = speed of light
$f_0$ = original frequency
$f_D$ = Doppler shift
$h_0$ = altitude constant
$m_1$, $m_2$ = coefficients of linear and parabolic components, respectively
$n$ = integrals of repetition frequency
$R$ = range
$T$ = period of waveform
$t_p$ = periodic time function defined by repetition rate desired
$V$ = velocity of aircraft
$V_0$ = velocity of observer
$V_s$ = velocity of source
$\Delta f_D$ = spectral bandwidth
$\delta$ = peak frequency deviation
$\gamma$ = antenna depression angle from horizontal
$\gamma_B$ = antenna beamwidth, half-power
$\lambda$ = wavelength of transmitted signal
$\omega_0$ = transmitted frequency

## Introduction

Workers in the Doppler radar field have for many years labored under the restrictions imposed upon them by the effects of physical antenna beamwidth. Unlike aircraft-type targets that may be considered point sources, the ground in a Doppler navigator completely fills the radar beam. The normal Doppler navigator return, then, far from being a monochrome, is rather a spectrum of frequencies of typically gaussian shape whose bandwidth is a function of velocity and antenna beamwidth. A typical Doppler ground spectrum is shown in Fig. 1.

Operational Doppler navigation radars are capable (presently) of providing ground speed information at velocities and altitudes in the region of 2500 fps and 60,000 ft, respectively. Doppler bandwidths, under these conditions, are usually on the order of a few kilocycles. At orbital velocities and altitudes (25,000 fps and 600,000 ft), however, this spectrum width increases to values in excess of 25 kc, demanding either an extremely complicated and wideband frequency tracker or an increase in the physical aperture of the antenna, which now begins to assume prohibitive size for present spacecraft. Either solution is unwieldy and/or unrealistic.

Furthermore, this spectral bandwidth will result in a loss of peak power amplitude (the power is spread over the entire spectrum, instead of being concentrated at one frequency), and it is the source of two severe regions of error, namely, fluctuation and terrain bias. For a space vehicle, power is of the essence, and the extremely high accuracies expected of a sensor of these applications demand that error sources be held to very fine tolerances.

With the need for a reasonable method of reducing spectral bandwidths in mind, a microscopic study was conducted. The study is termed microscopic because it considered the effects of any one scattering element on the ground, instead of the more general, macroscopic approach that considers a patch of ground with its multitude of scattering elements all acting in unison. The study concluded (and this constitutes the gist of this paper) that *the motion of each reflector could be modified* (with a resultant reduction in spectral width), *if the transmitted signal were modulated with a particular waveform.*[1, 2]

## Development of Continuous-Wave Radar Relationships

An understanding of the nature and mathematical relationships pertaining to continuous-wave (CW) Doppler radar will aid in following the development of spectral compression. In general, when the vibrating source of a train of waves is caused to move in the direction of the wave propagation, the effect to an observer is as follows:

$$f_0 \pm f_D = f_0[(C \pm V_0)/(C \mp V_s)] = f \qquad (1)$$

Now consider a geometrical configuration typical of practical Doppler navigators, shown in Fig. 2. In this instance, the aircraft is the source of propagation, and its velocity, along the antenna axis, is defined as $V_{s1}$. The observer is on the ground and has no velocity ($V_{01} = 0$). Substituting into Eq. (1) yields:

$$f_{D1} = f_0 V_{s1}/C \qquad C \gg V_{s1} \qquad (2)$$

This is the Doppler that is experienced on the ground. But the receiver is, of course, in the aircraft. Therefore, the ground now acts as a new source having zero velocity, and the

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aircraft is the new observer having a velocity $V$ that is equal to $V_{s_1}$, so that

$$f_2 = f_0 + 2f_{D_1} = f_0 + (2V_{s_1}/\lambda) \quad (3)$$

where $V_{0_2} = V_{s_1}$ and $\lambda = f_0/C$. Therefore, if the antenna is depressed at an angle $\gamma_0$ from the horizontal, the Doppler shift will be

$$f_D = 2V\,(\cos\gamma_0)/\lambda \quad (4)$$

The spectral bandwidth $\Delta f_D$ in a CW system is normally derived (in navigator applications) in any of several ways, e.g., a Taylor expansion about $\gamma_0$. Similarly, if one subtracted the Doppler frequency corresponding to the trailing edge of the beam in Fig. 2 from that of the leading edge, the result would constitute the spectral width of the radar signal. Both approaches would yield the same expression,[3]

$$\Delta f_D = (2V/\lambda)\,\sin\gamma_0(\gamma_B) \quad (5)$$

which has been proved countless times in actual operation. Still, the approach presented (which is the most commonly used method) is somewhat empirical in nature, since it treats the ground return from an infinitude of scattering elements as something of a mass phenomenon and, as such, glosses over the intrinsic motion of each individual scatterer as it passes through the radar beam in time. A more fundamental, implicit approach exists, however, which truly defines the development of the Doppler frequency shift (and its associated bandwidth), and that will be considered now.

In straight and level flight, the individual scatterer's geometrical relationships are as shown in Fig. 3. As the radar beam passes over the ground, the range to the $i$th scatterer will vary as a function of time:

$$R(t) = [h_0^2 + (a_i - Vt)^2]^{1/2} \quad (6)$$

Expanding Eq. (6) in a Maclaurin series yields a more convenient expression

$$R(t) = R_i - Vt\cos\gamma_i + \frac{V^2t^2\sin^2\gamma_i}{2R_i} + \frac{V^3t^3\sin^2\gamma_i\cos\gamma_i}{2R_i^2} + \ldots \quad (7)$$

For CW transmission, the transmitted and received waveforms are

$$e_T = E_T\sin\omega_0 t \quad (8)$$

$$e_R = E_R\sin\omega_0(t - \tau) \quad (9)$$

where the delay $\tau$ is $2R(t)/C$. Substituting Eq. (7) into

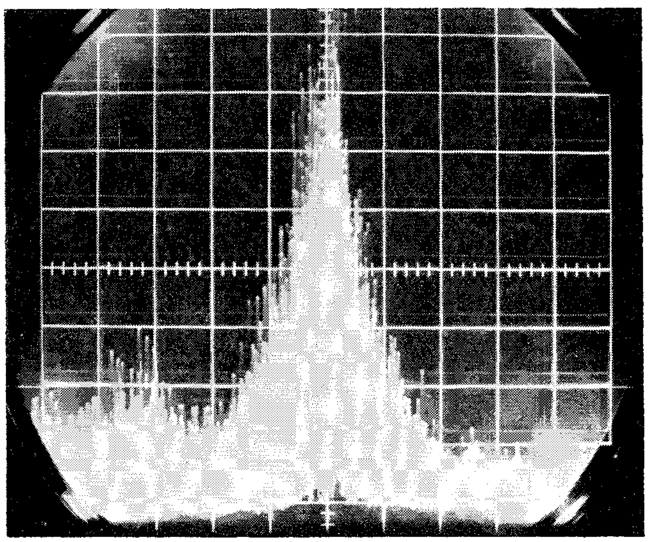

Fig. 1 Typical Doppler ground spectrum.

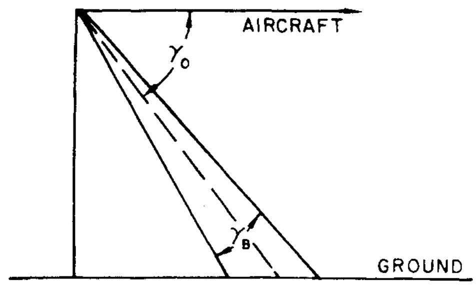


Fig. 2 Typical Doppler navigator geometry.

Eq. (9), and differentiating the phase of the resultant signal, will yield

$$\omega_R = \omega_0 + \frac{2\omega_0 V\cos\gamma_i}{C} - \frac{2\omega_0 V^2 t\sin^2\gamma_i}{R_iC} - \frac{3\omega_0V^3t^2\sin^2\gamma_i\cos\gamma_i}{R_i^2C} \quad (10)$$

where the last term may be dropped as insignificant. Another simple trigonometric manipulation, however, will reveal that the time required for a scatterer to traverse the radar beam completely is given as

$$t = h_0\gamma_B/V\sin^2\gamma_0 \quad (11)$$

For a scatterer at $\gamma_0$ ($\gamma_i = \gamma_0$), substitution of Eq. (11) into Eq. (10) and division by $2\pi$ yields:

$$f_R = f_0 + (2V/\lambda)\cos\gamma_0 - (2V/\lambda)\sin\gamma_0(\gamma_B) \quad (12)$$

But Eq. (12) reveals Doppler and bandwidth terms which are identical to those of Eqs. (4) and (5). Thus we have the line of reasoning that must be used in the development of the spectral compression technique.

## Mathematical Development of Spectral Compression

The problem facing the Doppler radar engineer, then, is what to do about the bandwidth term of Eq. (12). The first thing one notes in analyzing the normal Doppler situation is that the Doppler frequency corresponding to each individual scattering element in the radar beam bears nearly a linear relationship to that of each of the other reflectors at any instant. A reasonable first step, then, is to modulate the CW transmitter with a linear sawtooth, thus compensating for the different Doppler shifts to each of the multitude of random scatterers within the radar beam. This is, however, an instantaneous consideration; as time progresses, each individual reflector will itself assume every relative position in the radar beam; hence, one must compensate for this relative motion of the individual reflectors by *adding a parabolic com-*

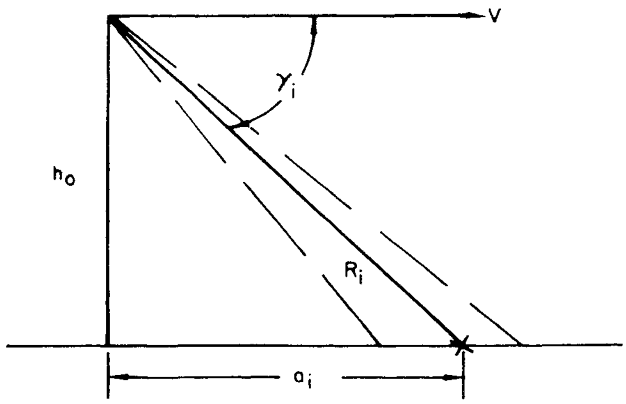


Fig. 3 Doppler scatterer geometry.

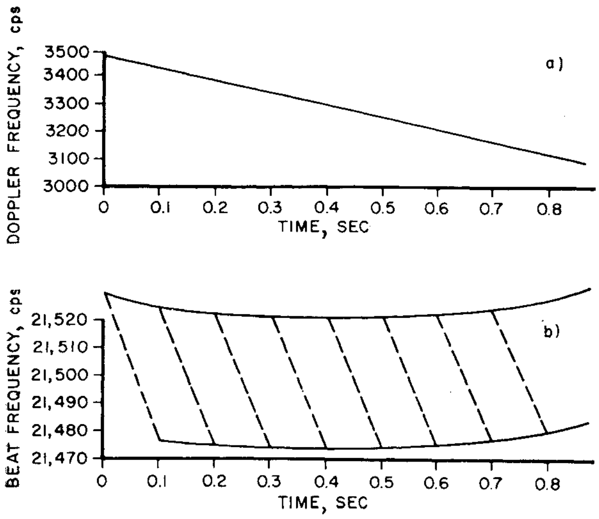


**Fig. 4 Frequency loci.**

*ponent to the linear sawtooth.* Therefore, the transmitted waveform for spectral compression is

$$e_T = E_T \sin[\omega_0 t_p + m_1 t_p^2/2 + (m_2 t_p^3/3)] \qquad 0 \leq t_p \leq T \quad (13)$$

As in CW, the returned signal will be delayed by $\tau = 2R(t)/C$:

$$e_R = E_R \sin[\omega_0(t_p - \tau) + (m_1/2)(t_p - \tau)^2 + (m_2/3)(t_p - \tau)^3] \quad (14)$$

But here one should differentiate between the periodic time function $t_p$ and the range function that is continuous throughout the radar beam. The range function of Eq. (7) should, therefore, be redefined as a continuous time function $t_c$ by substituting $t_c$ everywhere for $t$. The resulting range equation serves, in essence, to define the value of the depression angle $\gamma$ at any instant of time, whereas the periodic function of Eq. (14) serves to define the modulation process. The subsequent manipulations are straightforward and elementary but tedious; the reader is asked to accept the following result. Substituting the range equation into Eq. (14), mixing the returned signal with a portion of that transmitted, and differentiating the phase of the resultant signal will yield a signal having a beat frequency $f_B$ characterized as follows [this is the same procedure followed in developing Eqs. (8–11)]:

$$f_B = \underset{\text{(a)}}{-(2f_0V\cos\gamma_i/C)} + \underset{\text{(b)}}{(2f_0V^2t_c\sin^2\gamma_i/R_iC)} + \underset{\text{(c)}}{(3f_0V^3t_c^2\sin^2\gamma_i\cos\gamma_i/R_i^2C)} + m_1[\underset{\text{(d)}}{-(2Vt_p\cos\gamma_i/C)} + \underset{\text{(e)}}{(2V^2t_pt_c\sin^2\gamma_i/R_iC)} + \underset{\text{(f)}}{(2R_i/C)} - \underset{\text{(g)}}{(2Vt_c\cos\gamma_i/C)} + \underset{\text{(h)}}{(V^2t_c^2\sin^2\gamma_i/R_iC)}] + \underset{\text{(i)}}{m_2(4R_it_p/C)} \quad (15)$$

where one has conveniently omitted those terms that were calculated to be insignificant. *Equation (15) defines the instantaneous frequency return expected from the spectral compression technique.* The independent terms are labeled (a–i) so that they can be referred to in the material to follow.

As mentioned earlier, the aim of the linear modulation is to set the frequency return from each scatterer equal to that from any other scatterer at any instant of time. Consequently, the linear coefficient $m_1$ is determined by setting the time-independent terms (a) and (f) for a scatterer entering the beam ($\gamma_i = \gamma_0 - \gamma_B/2$) equal to those for a scatterer leaving the beam ($\gamma_i = \gamma_0 + \gamma_B/2$). Solving for $m_1$:

$$m_1 = f_0V\sin^3\gamma_0/h_0\cos\gamma_0 \quad (16)$$

This designation of $m_1$ serves, in essence, to fold the returns from the scattering elements (reflectors) on top of each other at one frequency to a first-order approximation.

The residual bandwidth (created by the relative motion of each individual reflector passing through the beam in time) can be further reduced by equating terms (b), (d), (g), and (i) of Eq. (15) to zero for a scatterer at $\gamma_i = \gamma_0$ and solving for the parabolic coefficient $m_2$:

$$m_2 = f_0V^2\sin^4\gamma_0/2h_0^2 \quad (17)$$

Under the conditions defined, the remaining higher-order terms ($t^2$, $t^3$, etc.) of Eq. (15) will constitute the resultant bandwidth terms of the compressed spectrum. For a scatterer at $\gamma_i = \gamma_0$ then, Eq. (15) can be reduced to

$$f_B = -\frac{2V}{\lambda}\left(\frac{\cos2\gamma_0}{\cos\gamma_0}\right) + \frac{V^3t_c\sin^4\gamma_0}{h_0\lambda^2} \times \left[3t_c\cos\gamma_0 + \frac{\sin^2\gamma}{\cos\gamma_0}(2t_p + t_c)\right] \quad (18)$$

where the first term of the right-hand member is the new median of the spectrum (from which velocity is calculated directly), and the remaining terms represent the bandwidth of the spectrum.

## Numerical Analysis of Spectral Compression

Some practical limitations and tradeoffs that must be made to effect a workable system should be mentioned before we proceed to the numerical evaluation. The approach presented is based on the instantaneous frequency concept and leans heavily on the geometric relations of Fig. 3. However, the system is periodic and must obey the rules of Fourier analysis. This being the case, the effects of periodicity can be summarized quite simply (a more detailed analysis of the resultant spectral shapes is presented later). The geometric approach is valid only if the repetition rate is low enough so that the spectral lines are close to the predicted geometrical value of beat frequency (energy can exist only in these sidebands, regardless of the predicted geometrical value). If the repetition frequency is high compared to the Doppler bandwidth, the predicted results will not be achieved. Beat frequency can occur only at integrals of the repetition frequency. As a result, *the normal Doppler bandwidth can be reduced in the limit to a value no less than that of the repetition frequency.* If spectral compression is to be effected, one must operate at very low repetition frequencies.

Examination of Eq. (15) reveals that the technique depends upon simultaneous range-Doppler discrimination. To be successful, then, one must maintain very high range resolution.[4] This problem has been carefully analyzed, and the range resolution can be represented in the following simple form:

$$\Delta R > nC/2\delta \quad (19)$$

To insure that the range lines remain truly distinct, the resolution in frequency should be maintained at about three times the repetition rate ($n = 3$). One sees now that *ultimate compression is also dependent upon the maximum deviation possible with a given transmitter.* In essence, then, if the total range difference, from one edge of the beam to the other, were, say, 100 ft and the range resolution were 10 ft, the maximum compression ratio would be limited to 10:1.

The specific objective for this technique is utilization in a boost glide or other orbital vehicle But in an attempt to prove the validity of the theory, a flight-test program was conducted with a B-26 aircraft.

Typical conditions were: $V = 350$ fps, $h_0 = 6000$ ft, $t_p = 0.1$ sec, $\gamma_0 = 70°$, $\gamma_B = 2° \; 33'$, and $f_0 = 13.5$ kmc.

With these quantities, Eq. (4) yields the nominal value of the Doppler frequency shift in a CW system, i.e.; $f_D = 3286.3$ cps, and an antenna beamwidth of $2° \; 33'$ in Eq. (5) gives a CW spectral bandwidth of $\Delta f_D = 401.7$ cps. This means that every scatterer enters the radar beam at an angle of $68° \; 43.5'$, leaves at an angle of $71° \; 16.5'$, and in the process takes on corresponding Doppler frequencies ranging from 3486.4 to 3084.7 cps. The loci of the frequencies is shown in Fig. 4a.

For spectral compression, substitution of the required quantities into Eqs. (16) and (17) will yield the modulation coefficients necessary for proper operation, i.e., $m_1 = 1.91055 \times 10^9$ cps$^2$, and $m_2 = 1.79092 \times 10^7$ cps$^3$.

The expected results are summarized in Fig. 4b. Using both linear and parabolic modulation, the instantaneous beat frequency will vary as the upper curve, going from 21,529.29 cps at the leading edge of the radar beam to a minimum value of 21,520.61 cps near the center, and back up to 21,530.51 cps as the scatterer leaves the beam. This represents a maximum change (bandwidth) of 9.90 cps, as compared to the CW bandwidth of 401.7 cps, i.e., a compression ratio of better than 40.

If only linear modulation is used (set $m_2$ to zero in Eq. 15), the beat frequency behaves as follows. During each modulation period, the beat frequency will vary from that of the upper curve of Fig. 4b to that of the lower curve, and then will return instantaneously to the upper curve as the waveform is reset. The resultant beat frequency is then another sawtooth but one whose total frequency excursion is much less than that of CW. In this instance, for example, the maximum excursion is that from the leading edge of the beam (21, 529.29) to a minimum of 21,474.60. This represents a change of 54.7 cps and a compression ratio of 7.3 to 1 that is still significant. Therefore, although both linear and parabolic modulations are necessary for maximum compression, the use of linear modulation alone will still result in significant compression. The decision to add parabolic modulation (with its increased complexity) must be based on the needs of a particular application.

Again, these considerations are based on instantaneous frequency. Recalling that the modulation period was chosen to be 0.1 sec, the FM sidebands would occur at 10 cps (as opposed to the 9.90 cps calculated above). Even more significant, however, is the limitation imposed by range resolution. At an altitude of 6000 ft, the range along the leading edge of the beam is greater than that along the trailing edge by a mere 113.5 ft. The peak deviation in this situation is defined by

$$\delta = m_1 t_p = 1.91055 \times 10^9 \times 0.1 = 1.91055 \times 10^8 \text{ (cps)} \quad (20)$$

Substituting this quantity into Eq. (19) will reveal a range resolution of $\Delta R \approx 7.7$ ft. On this basis, one could compress the spectrum by no more than the ratio of 113.5 to 7.7, or approximately 15. This would then represent the limiting factor for the conditions assumed. In the boost glide (orbital) case, however, the range resolution would remain reasonably constant while the range difference along the leading and trailing edges of the beam would, of course, increase significantly. Therefore, range resolution should not be a significant factor in high-altitude situations.

A similar analysis has been conducted for typical orbital conditions: $V = 25{,}000$ fps, $h_0 = 600{,}000$ ft, $t_p = 0.05$ sec, $\gamma_0 = 70°$, $\gamma_B = 2° \; 6'$, and $f_0 = 13.5$ kmc. Under these conditions the CW bandwidth is calculated to be 23,711.8 cps. *Using both linear and parabolic modulation, the maximum variation in beat frequency is 471.6 cps, representing a compression ratio of better than 50* on an instantaneous frequency basis. If only linear modulation were used, the maximum bandwidth would increase to 1592 cps, and the compression ratio would still be approximately 15.

Unlike the flight-test situation, repetition frequency and range resolution would no longer be limiting factors. Assuming a modulation period of 0.05 sec, the sidebands would occur every 20 cps. But the smallest bandwidth attainable is 471.6 cps and is, therefore, not affected. The peak transmitter deviation required is

$$\delta = m_1 t_p = 1.36466 \times 10^9 \times 0.05 = 6.82330 \times 10^7 \text{ cps} \quad (21)$$

The resulting range resolution is $\Delta R \approx 21.6$ ft, but the differential range is now 8090 ft, which gives a spectral compression of approximately 375. Range resolution is, therefore, insignificant.

Summarizing the numerical analysis, one should reasonably expect to attain compression ratios in the order of 50 under the orbital conditions assumed. Since peak power is directly related to bandwidth, one should expect it to increase by a factor of 50. Fluctuation error, due to the noise-like nature of the spectrum, is a function of the square root of bandwidth and should, therefore, be reduced by the square root of 50, approximately seven times. Terrain bias error that results from the slope of the earth's reflectivity coefficient and that is a function of the square of the beamwidth in the $\gamma$ direction (which has been artificially reduced by a factor of 50), is, for practical purposes, eliminated.

## Development of Resultant Spectral Shapes

With a CW system, one can always predict a Gaussian spectrum such as that shown in Fig. 1, because of the high time-bandwidth product of the system (the product of the time of passage of one scatterer through the beam and the resultant bandwidth of the spectrum). The fact is, however, that the Fourier transform of a CW signal will yield an amplitude-frequency spectrum that is rectangular, in the absence of antenna weighting. The resultant Gaussian spectral shape is due almost entirely to the Gaussian antenna power pattern. In other words, if the antenna power pattern were constant rather than Gaussian, the CW spectrum would be rectangular. However, in a spectral compression system, wherein the time-bandwidth product can become quite small, one cannot automatically presume a rectangular amplitude-frequency distribution. The actual shape of the compressed spectrum can only be found by the Fourier transform.[5]

Consideration of Eq. (15) reveals that, whereas the scattering (reflecting) element passes through the beam as a continuous time function $t_c$, the modulation waveform is reset periodically as a function of $t_p$. In the absence of parabolic modulation, then, solution of Eq. (15) on an instantaneous frequency basis yields beat frequency waveforms (as seen earlier) that start at a maximum frequency $\omega_{max}$ and that decrease linearly to a minimum frequency $\omega_{min}$ during any period of $t_p$ sec. However, the maximum and minimum frequencies of each sawtooth will differ on each successive period, because the loci of these frequencies is parabolic. The waveform is, therefore, aperiodic. Each sawtooth may then be characterized by its center frequency $\omega_c$ and its slope $\mu$. The time-voltage waveform for each sawtooth may now be written as

$$E(t) = A(t) \cos(\omega_c t + \tfrac{1}{2}\mu t^2) \quad (22)$$

where

$$-(t_p/2) \leq t \leq t_p/2 \quad (22)$$

Each voltage sawtooth throughout the radar beam will generate an independent amplitude-frequency spectrum,

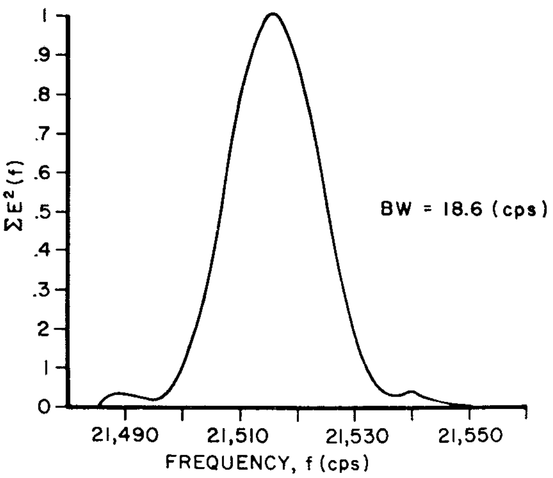


**Fig. 5 Composite power-frequency spectrum for 20 cps sawtooth, TBW ≈ 1.**

whose characteristic shape and bandwidth will be a function of the product of the sawtooth period and the instantaneous frequency excursion. Each amplitude-frequency spectrum may be determined by solution of the Fourier integral:

$$E(\omega) = \frac{1}{2}\int_{-(tp/2)}^{tp/2} A(t) \exp\left\{ j \left[ (\omega_c - \omega)t + \frac{1}{2}\mu t^2 \right] \right\} dt \quad (23)$$

The function $A(t)$ in this instance is the antenna weighting function which is defined as an exponential as follows:

$$A(t) = \{\exp[-KV^2 \sin^4\gamma_0(t_x + t)^2/h_0^2]\}^{1/2} \quad (24)$$

where $t_x$ is a constant and defines the length of time that the scatterer has been in the radar beam when $t = t_p/2$, and $K$ is the constant that sets the exponential to 0.5 at the spectrum's half-power points. Substitution of Eq. (24) into Eq. (23) yields the equation that must be solved to determine the amplitude-frequency spectrum of any given sawtooth. However, the significant parameter is the total effect of all the spectra generated by all of the reflectors in the radar beam. An excellent estimate of this sum may be obtained by deriving the spectra associated with each sawtooth which any one reflector will generate in traversing the radar beam, squaring the voltage spectra to power spectra, and adding all of the resulting spectra to form one composite spectrum.

Equations (22–24) were programed on an IBM 704 digital computer, and a complete Fourier analysis was performed to determine the precise spectral shapes that should be expected for a number of typical applications. The situation characterized in the numerical analysis yielded a bandwidth (for linear modulation alone) of 54.7 cps, which far exceeded the repetition frequency of 10 cps. In an effort to demonstrate the limiting effect of the repetition frequency (an AM effect), let us change the conditions of the numerical analysis slightly, raising the altitude to 10,000 ft and the repetition frequency to 20 cps ($t_p$ = 0.05 sec). Under these conditions, each individual sawtooth would have a frequency excursion of some 13 or 14 cps and a total spectrum width of approximately 20 cps. This represents a time-bandwidth product (20 × 0.05) of approximately 1.0.

Under these conditions, a reflector would require approximately 1.45 sec to traverse the radar beam, and 29 separate sawteeth would be generated. Solution of Eq. (23) reveals that each sawtooth would have a voltage-frequency spectrum that is peaked at the center and demonstrates a typically $\sin x/x$ characteristic, which is what should be expected of a waveform having a time-bandwidth product approximating unity. Each of the spectrum shapes generated by the 29 sawteeth were determined, their voltage spectra were squared to yield power spectra, and all of the power spectra were summed to yield a composite power-frequency spectrum. The result is shown in Fig. 5 and constitutes the normalized representation of a beamwidth containing an infinitude of scattering elements. The bandwidth at the half-power points is seen to be approximately 18.6 cps, as opposed to a CW bandwidth of approximately 400 cps for the same conditions, representing a compression of better than 20 to 1. Now, elementary modulation theory dictates that for a modulation rate of 20 cps, sidebands should occur at every 20 cycles, and, therefore, bandwidth should be no less than this. In essence, however, the spectral bandwidth is a matter of definition, here defined as that width occurring at the spectrum's half-power points and, as such, is purely arbitrary. The total energy spectrum is, of course, at least as wide as the sideband structure, and this is readily seen in Fig. 5 by observing the width of the spectrum at its base.

At this modulation rate (20 cps), however, one difficulty exists. The instantaneous beat frequency excursion for each sawtooth in the beam (in the idealized situation) is of the order of 13 or 14 cps, which is less than the modulation rate. As a result, the AM effect is greater than the FM and dominates the shaping of the spectra. Decreasing the modulation rate ultimately will decrease the AM effect to one of insignificance. However, the residual bandwidth due to the FM will increase, as evidenced by the results of the numerical analysis. One consolation is that the problem exists only in the low-velocity B-26 flight condition. In the boost-glide application, even ultimate compression will have a bandwidth far in excess of that created by amplitude modulation, and, therefore, the residual bandwidth will be independent of the repetition frequency.

For the B-26 case, if one reduces the modulation rate to 5 cps, a residual bandwidth of approximately 50 cps will result. The voltage-frequency spectrum for any sawtooth in this instance is shown in Fig. 6. The time-bandwidth product here is of the order of 10.

In the boost-glide application, the Fourier transform (for this case) would yield the voltage-frequency spectrum shown in Fig. 7. The time-bandwidth product here approximates 50. The spectrum is relatively rectangular and begins to approach a CW shape. The spectra of Figs. 6 and 7 were plotted for a scatterer at the center of the radar beam.

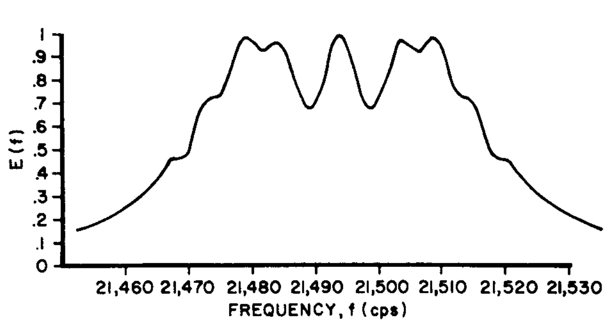


**Fig. 6 Voltage-frequency spectrum for 5 cps sawtooth, TBW ≈ 10.**

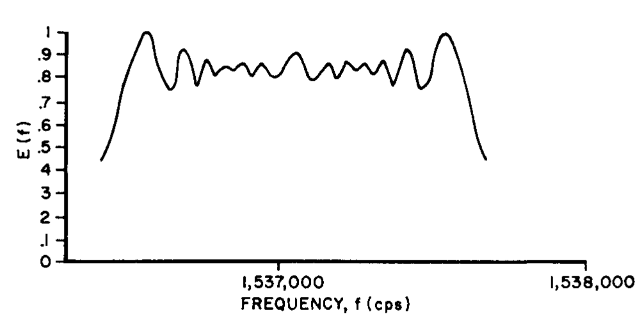


**Fig. 7 Voltage-frequency spectrum for 20 cps sawtooth, TBW ≈ 50.**

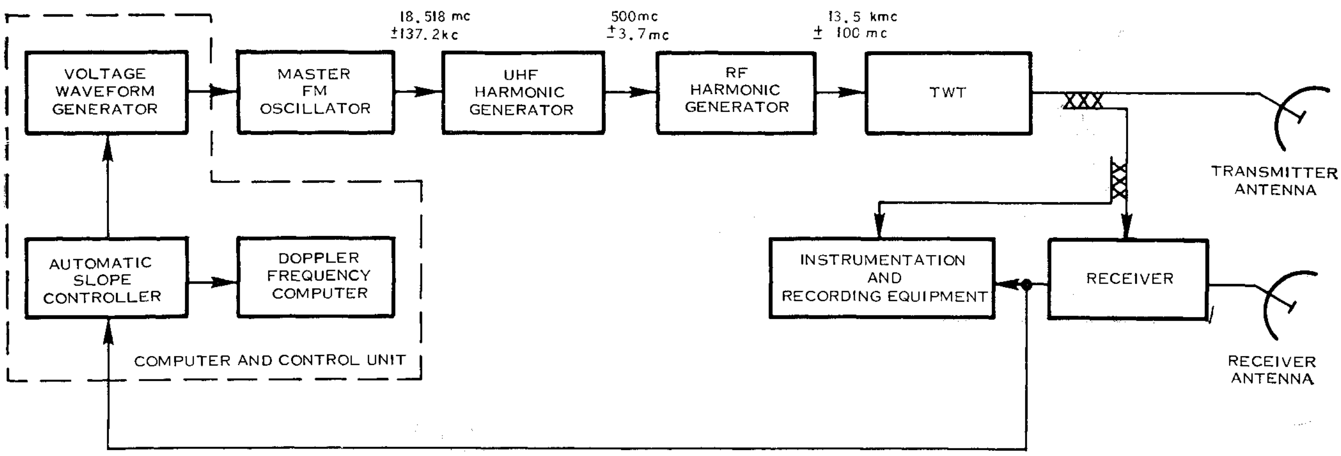


**Fig. 8 Spectral compression Doppler sensor.**

## Equipment

Figure 8 shows a simplified block diagram of the experimental spectral compression system and instrumentation, including the automatic slope-control loop and a Doppler frequency computer, neither of which were included in the flight-test program. The basic pulse-repetition frequency (PRF) timing waveform originates in a small, lightweight, stable, commercial audio oscillator. Pulse-shaping circuits provide the necessary reset commands to the linear and parabolic waveform generators that consist of stable resistance-capacitance networks and high-gain operational amplifiers, which together develop precise voltage waveforms. These three elements form the voltage waveform generator.

Linear FM modulation is generated at a low frequency (approximately 9.259 Mc) by application of the voltage waveform to the voltage-controlled oscillator (VCO). Open loop, the VCO has a nearly linear frequency-vs-voltage characteristic. Closing the loop with a linear frequency-to-voltage converter in the feedback network forces the VCO to be linear within 0.02%. Multiplication by 2 raises the FM signal to approximately 18.518 Mc. Heterodyning the 9.259 Mc signal with a crystal oscillator at 8.55 Mc produces an FM signal at 709 kc for the linear frequency-to-voltage converter. Three triplers raise the signal from 18.518 Mc to the order of 500 Mc. Applying the 500 Mc signal to a varactor, rf harmonic multiplier produces a signal at 13.5 kMc, along with the required deviation, 200 Mc. A waveguide filter is used to reject the undesirable harmonics. The 13.5 kMc signal must then be raised to the required power level in a traveling-wave tube (TWT) amplifier. The output of the TWT is now at a suitable power level (7 w) to provide sensitivity, and contains the required peak frequency deviation.

Space-duplexed, FM/CW transmission is utilized, and the antenna is pointed aft along the aircraft centerline at a depression angle of 70°. A homodyne (zero frequency i.f.) receiver was chosen to avoid the development of a wideband, single-sideband generator that would have to maintain good carrier and unwanted sideband rejection over an AM deviation of up to 200 Mc. The beat frequency signal from the video amplifier is recorded on tape for further ground evaluation and can be viewed on the 3 kc analyzer mounted in the aircraft.

## Experimental Flight-Test Results

The test results were most gratifying. In fact, the problem of significance at the time was to determine just how good the results had been. This problem arose from the following:

1) The system was controlled manually so that an optimum modulation slope could be applied at any one instant. But if any conditions (e.g., velocity, pitch, roll, etc.) changed slightly, the beat frequency would change, and compression would no longer be optimum. As a result, near theoretical compression could be observed during any one (or series of) modulation period(s), whereas integration over any significant period of time would yield a result that was suboptimum (although still significantly compressed). (A system to detect any increase in spectral bandwidth and automatically to modify the modulation slope was designed and laboratory tested; unfortunately, the program ended before the automatic system was thoroughly evaluated in flight.)

2) The spectral information was viewed on a Rayspan, rod-type spectral analyzer whose filter bandwidth is of the order of 10 cps/rod. However, if two signals are to be truly distinguishable, their frequency separation must be approximately 25 cps or more. This constitutes an equipment resolution problem.

3) Unlike the CW case, in which the Doppler frequencies are of the order of 2 to 3 kc, the beat frequency spectrum for spectral compression will generally appear in the region of 20 kc (for B-26 velocities). At these frequencies, flutter and wow from the tape-recording equipment become excessive (e.g., at 17 kc smearing amounted to about 40 cps, whereas at 30 kc, as much as 130 cps of smearing was measured). During the flight test, then, the procedure was altered such that one heterodyned the Doppler signal with a variable oscillator to produce an output in the frequency range of 2 to 3 kc which reduced tape flutter and wow to about 5 cps. In this manner, the narrow bandwidths achieved in flight may be truly viewed on the ground. Despite the problems stated, the experimental results testify to the validity of the theoretical models and analytical approaches assumed. For example, Fig. 9 shows a CW return at 8000 ft over water, a ground speed of 210 knots (354 fps), a Doppler frequency of 3350 cps, and a Doppler bandwidth of 360 cps (approximately a 10.7% bandwidth). The analyzer sweep rate is 100 cps per grid division, and peak spectral density amplitude is about 1.2 grid divisions high. The addition of both linear and parabolic modulation at a PRF of 15 cps will yield a compressed spectrum centered at 20.8 kc (Fig. 10). The compressed bandwidth is approximately 40 cps, representing a bandwidth reduction of 9 to 1.

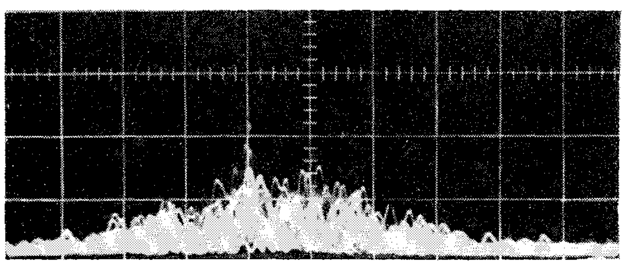

**Fig. 9 CW spectrum, flight tape data.**

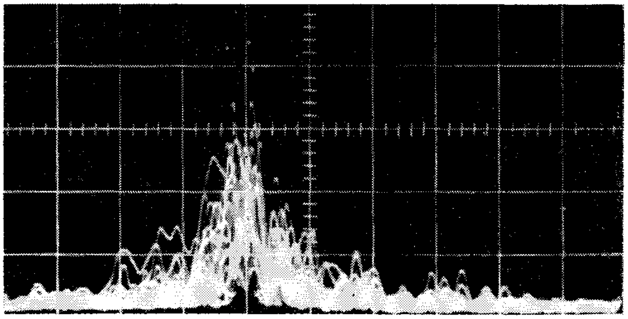

Fig. 10 Compressed spectrum, flight tape data.

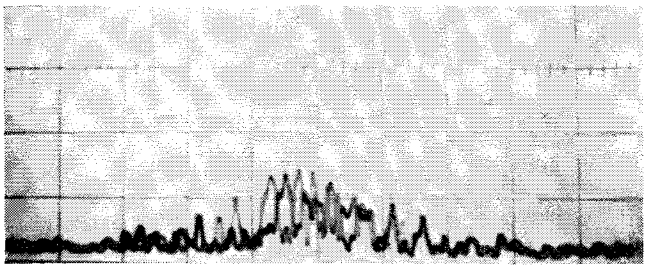

Fig. 11 CW spectrum, film data.

The peak spectral density amplitude is approximately 4.2 grid divisions high (voltage), representing a power increase of better than 12 to 1, which agrees fairly well with the bandwidth reduction.

The results indicated by Figs. 9 and 10 are, therefore, somewhat pessimistic despite the 9 to 1 reduction. Theoretical predictions are further validated, however, by consideration of the following. A motion picture camera was set at 10 frames/-sec (to coincide with the PRF of 10 cps) to record, in flight, the actual events occurring during any one modulation period.

Figure 11 represents CW transmission at 8000 ft over water at a velocity of 190 knots and a Doppler frequency of approximately 2900 cps. The scale is 200 cps per grid division. Comparing Fig. 11 with Fig. 9 reveals that they are quite similar in all respects (bandwidth, amplitude, etc.). Therefore, frame-by-frame analysis of the movie film reveals identical CW characteristics, whether viewed momentarily or integrated over one second. However, the compressed spectrum of Fig. 12 (with both linear and parabolic modulation) shows a compression far better than the 9 to 1 accomplished in Fig. 10. In fact, one single rod of the analyzer is seen to be energized, revealing a total bandwidth at the 3 db (0.707) points of less than 25 cps. The bandwidth is probably significantly lower than the 25 cps, because as the value of 25 cps was approached, another rod would become visible, even though of lesser amplitude. Use of the automatic slope-control system will tend to eliminate the discrepancies cited here.

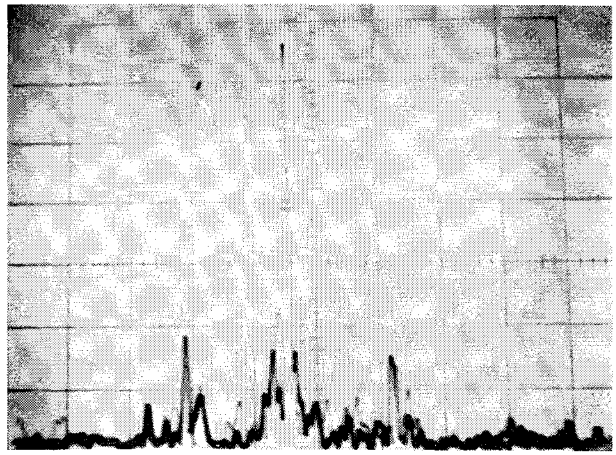

Fig. 12 Compressed spectrum, film data.

The results presented here are those obtained primarily over water, where the physical conditions more closely approximate the theoretical ideal of a flat, smooth earth. Since waves on a typical day are of the order of a few feet, the range discrepancies from one reflector to another represent a small percentage of the total range. Tests have been made over all kinds of terrain, however, and the results over reasonably smooth land are nearly as good as those over water. Unfortunately, when the terrain becomes very choppy (e.g., over cities), the compression degrades. At orbital altitudes, again, the effect of these range discrepancies should be decreased.

The true significance of these results goes beyond the application of spectral compression to the ends cited in the contract objective (more power, less error, etc.), because one has now truly defined the Doppler-scatterer relationship and found that the spectral return can be altered and manipulated by the application of particular modulation waveforms. The use of this technique toward the accomplishment of other similar goals is now merely a matter of modification.

## References

[1] Berger, F. B., "Automatic ground position indicator," General Precision Laboratories Interim Engineering Rept A11-17, 11-1-11-9, (January 1948).

[2] Price, R. and Green, P. E., Jr., "Signal processing in radar astronomy-communication via fluctuating multipath media," Lincoln Lab. Tech. Rept. 234, C-60 (October 6, 1960).

[3] Berger, F. B., "The nature of doppler velocity measurement," IRE Trans. Aeronaut. Navigational Electron., Vol. ANE-4, 103-112 (September 1957).

[4] "Some aspects of Doppler methods for aircraft speed measurement at close range—0 to 500 feet," Royal Aircraft Establishment, Bedford, England, AD236538 (January 1960).

[5] Cook, C. E., "Pulse compression—key to more efficient radar transmission," *Proceedings of the IRE International Convention* (Institute of Radio Engineers, New York, March 1960), pp. 310-316.